

### ABSTRACT

Generalized Maxwell distribution law is used to explain some empirical relations concerning neutron scattering. These relations are concerned with the neutron and photon scattered fluxes in relation with wave lengths, flux path and energy. The relations are also concerned with the relation of mean free path with atomic number beside relation between fusion probabilities with neutrons fluxes. All these empirical relations are explained on the basis of generalized Maxwell distribution adequately.

**KEYWORDS:** generalized Maxwell distribution, intensity, mean free path, energy

### I. INTRODUCTION

Statistical physics is one of the most widely used theories in describing the behavior of huge number of particles that exist in bulk matter, condensed matter and radiation beams[1,2,3]. There are three main distribution Laws, Maxwell-Boltzmann, Bose- Einstein and Fermi-Dirac[4,5]. Gibbs distribution law[6]. These statistical Laws can success fully describe the behavior of some phenomena associated with bulk matter[7,8]. Especially those which are related to thermal behavior[9].But unfortunately those Laws fail in describing some recently new phenomena, like thermal properties of super conductors. Even nuclear decay Law and collision probability cannot be described by them.This motivates some authors to construct new generalized statistical model based on plasma equations. The main advantage of this model is related to considering the average energy (beta) to be not restricted to thermal energy only but includes other energy forms[10,11,12]. This encourages trying to describe scattering of light and neutrons by using this model.

### II. GENERALIZED MAXWELL LAW FOR INTENSITY RELATIONS

The relation between optical absorption I and the wave length of Can be found by using Generalized Maxwell Distribution (GMD) relation[10].

$$I = I_0 e^{-\frac{E}{\bar{E}}} \quad (1)$$

Where one used the relation and

$$\bar{E} = E_0$$

$$n = n_0 e^{-\frac{E}{E_0}} \quad (2)$$

Together with the fact that

$$I = nv$$

Treating a photon or neutron wave obeying Plank and De Broglie quantum relation the average energy is thus given by

$$E = hf = \frac{hc}{\lambda} \quad (3)$$

Inserting (3) in (1) yields

$$n = n_0 e^{-\frac{E}{hc}\lambda} \quad (4)$$

Thus equation (19) reads

$$I = I_0 e^{-\frac{E}{hc}\lambda} \quad (5)$$

This relation is displayed graphically in figure (1). The experimental work done by T. Abo-Shdeed, M. Nahili and I. Shaaban [1] show the empirical relation for neutron (photon) scattering in figure (2) which conforms with theoretical one in figure (1)

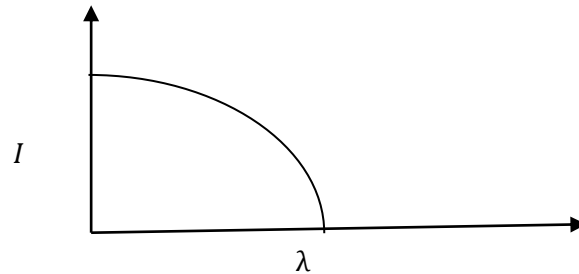


Fig (1) Theoretical relation between I and λ

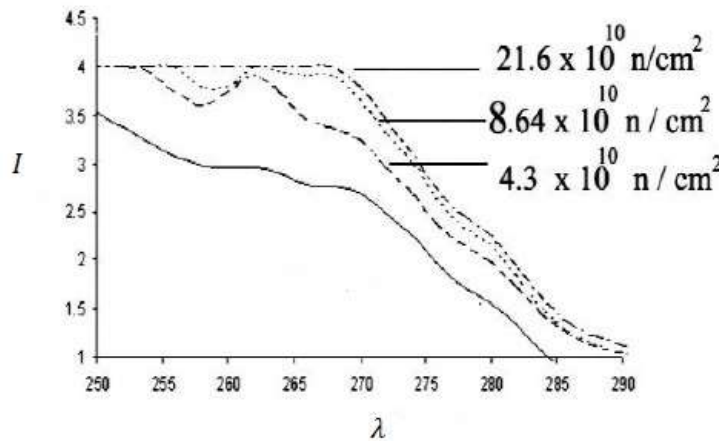


Fig (2) Empirical between I and λ for photons

The relation between photon intensity (production) and path length can be explained by using the expression of the average photon

$$\bar{E} = \frac{E_1 + E_2 + \dots + E_N}{N} = E_0 \quad (6)$$

The photon intensity is by

$$I = nc = n_0ce^{-\frac{E}{E_0}} = I_0e^{-\frac{E_p}{E_0}} \quad (7)$$

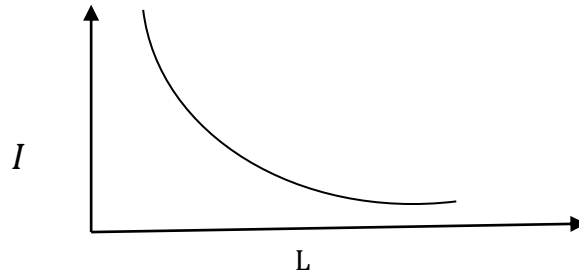


Fig (3) Theoretical relation between I and L for photons

Table (3): Empirical relation between (neutron, photon) production and thickness process  $^9\text{Be}(p,n)^9\text{B}$

$E_p=15\text{ MeV}$					
(photon/neutron)	Error %	(photon/sec)	Error %	(neutron/sec)	thickness cm
12.7235	0.48%	$3.315 \times 10^{12}$	1.67%	$2.606 \times 10^{11}$	0.0005
0.0390	2.59%	$3.071 \times 10^{11}$	0.51%	$7.864 \times 10^{12}$	0.5
0.0390	2.58%	$3.066 \times 10^{11}$	0.51%	$7.869 \times 10^{12}$	1.0
0.0389	2.56%	$3.064 \times 10^{11}$	0.50%	$7.884 \times 10^{12}$	1.50
0.0388	2.57%	$3.060 \times 10^{11}$	0.50%	$7.881 \times 10^{12}$	2.0
0.0390	2.55%	$3.076 \times 10^{11}$	0.50%	$7.887 \times 10^{12}$	2.50
0.0389	2.58%	$3.065 \times 10^{11}$	0.51%	$7.877 \times 10^{12}$	3.0
0.0389	2.58%	$3.063 \times 10^{11}$	0.51%	$7.878 \times 10^{12}$	4.0
0.0388	2.57%	$3.058 \times 10^{11}$	0.51%	$7.879 \times 10^{12}$	5.0
0.0387	2.57%	$3.056 \times 10^{11}$	0.50%	$7.889 \times 10^{12}$	6.0
0.0387	1.83%	$3.059 \times 10^{11}$	0.36%	$7.897 \times 10^{12}$	7.0
0.0388	2.55%	$3.063 \times 10^{11}$	0.50%	$7.903 \times 10^{12}$	8.0
0.0388	2.57%	$3.063 \times 10^{11}$	0.51%	$7.900 \times 10^{12}$	9.0
0.0390	2.57%	$3.07 \times 10^{11}$	0.51%	$7.852 \times 10^{12}$	10.0

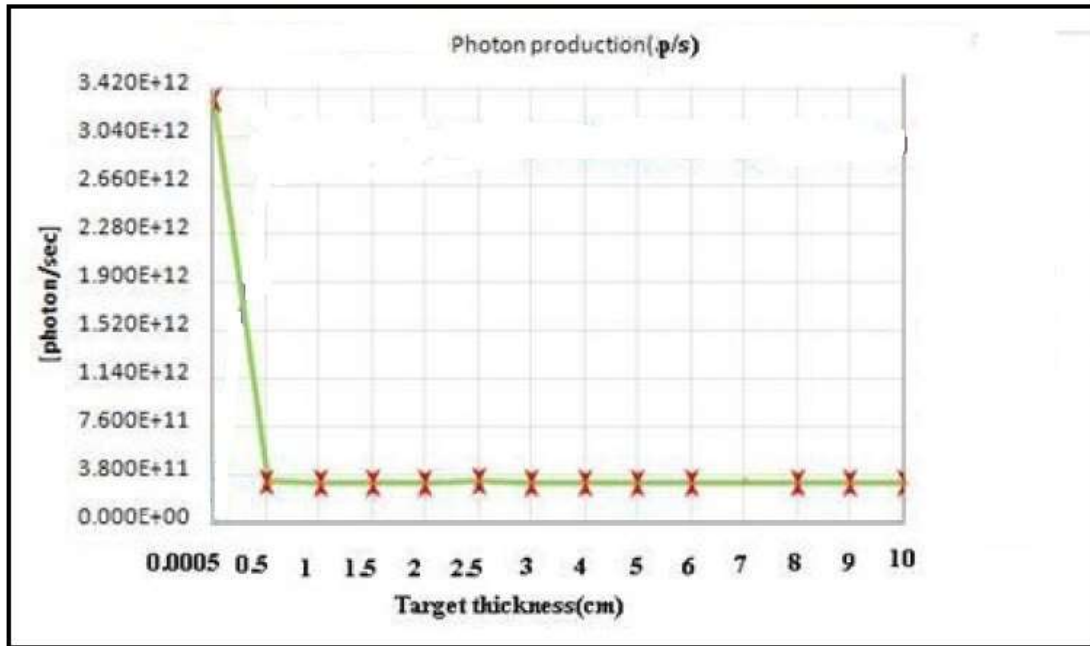


Fig (4) Empirical relation of photon production and target thickness

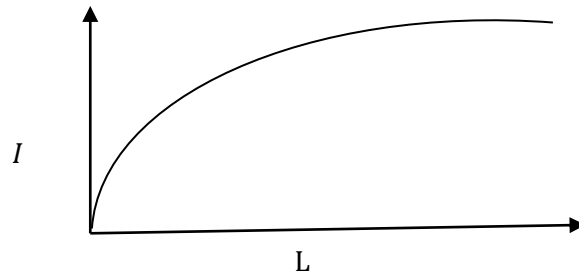


Fig (5) Theoretical relation between neutron production and thickness neutron magnetic

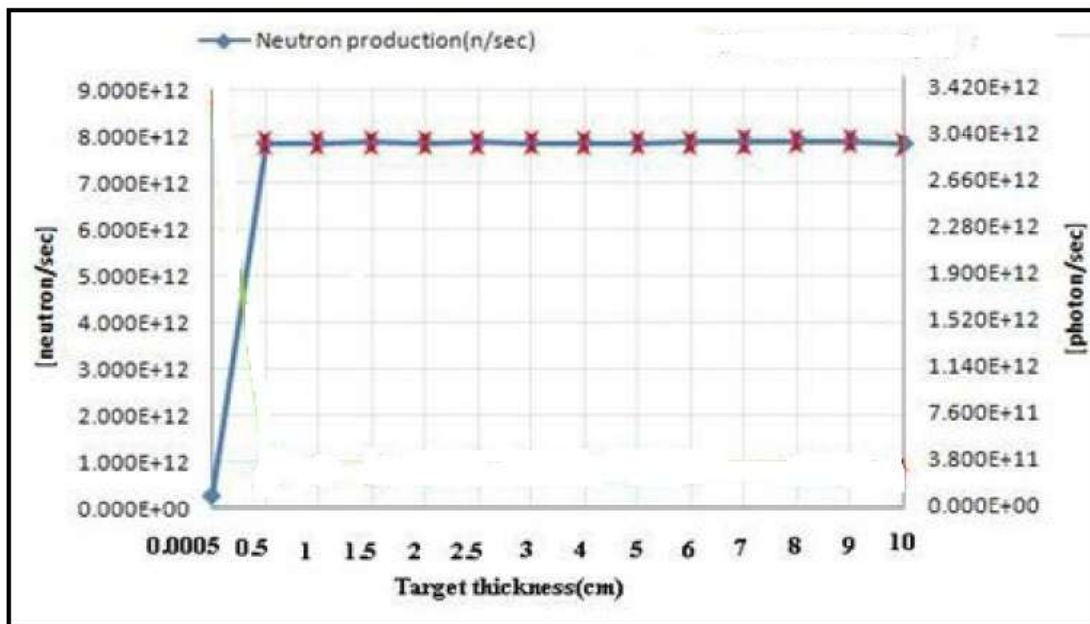


Fig (6) Empirical relation of neutron yield and target thickness in <sup>9</sup>Be (p,n)<sup>9</sup>B scattering

When neutral atoms are excited then each atom gives energy  $E_a$  to the photon. Thus the final photon energy is the sum of initial energy  $E_i$  and the energy given by all atoms on the path of the photon which have length  $L$ , where the number of atoms per unit length is  $n_0$ . thus the final photon energy is given by

$$E_p = E_i + nE_a = E_i + (n_0L)E_a \quad (8)$$

The total number of atoms within the path length  $L$  is given by

$$n = n_0L \quad (9)$$

The fact that the photon can gain energy from atoms resembles that of Raman scattering process, where one have stocks and anti stocks energy shift. Thus using equation (7) and (8) the photon intensity given by

$$I = I_0 e^{\frac{E_i}{E_0}} e^{\frac{-n_0E_aL}{E_0}} \quad (10)$$

This relation is displayed graphically in figure (3). This relation in figure (3) conforms to the empirical one in figure (4).

For neutrons having  $n_{0n}$  atom per unit length. The total number for the whole path is given by

$$n_n = n_{0n}L \quad (11)$$

The neutron having magnetic moment  $\mu_n$  moving in a medium having magnetic flux density  $B$  will gain magnetic potential

$$V_m = \mu_n B \quad (12)$$

The magnetic flux density, here is assumed to result from all atoms along the neutron path, therefore

$$B = B_n n_n = B_n n_{0n}L \quad (13)$$

Hence the energy of the neutron is given by

$$E = k + V_m = k + n_{0n}B_n\mu_n L \quad (14)$$

Where  $k$  is the neutron kinetic energy.

Assuming the average neutron energy to be due to the magnetic attraction between neutron magnetic moment and the magnetic flux density of the bulk matter one can write

$$\bar{E} = -V_0 = -E_0 \quad (15)$$

Where one neglects kinetic energy by assuming potential energy to be strong. Thus the intensity is given by

$$I = nv = nv_0 e^{\frac{E}{\bar{E}}}$$

$$I = I_0 e^{\frac{n_{0n}B_nL}{E_0}} \quad (16)$$

This relation (16) is displayed graphically in Fig (5). Fortunately this curve resembles the empirical one for the relation between neutron production and the target thickness in fig (6). The relation between mean free path  $\lambda$  and mass number  $A$  which was found empirically can be found theoretically by assuming very strong magnetic interaction between neutron magnetic moment  $\mu_n$  and nuclear magnetic flux density  $B$  where

$$E = V_m = \mu_n B = \mu_n \frac{\mu_0 Z e f}{2r} \quad (17)$$

Here one assumes  $Z$  proton revolve around nucleus centre with average radius  $r$

With frequency  $f$ , to have magnetic flux density

$$B = \frac{\mu_0 Z e f}{2r} \quad (18)$$

If the average magnetic energy is strong and attractive

$$\bar{E} = -V_0 \quad (19)$$



Hence the mean free path  $\lambda$  which depend on matter density  $\rho$  and cross section  $\sigma$  is given by

$$\lambda = \frac{1}{\rho} \sigma^{-1} = \frac{1}{\rho} N_{sc}^{-1} = \frac{n_0^{-1}}{\rho} e^{+\frac{E}{E}} \quad (20)$$

Where

$$\sigma \sim N_{sc} \sim n_0 e^{\frac{E}{E}}$$

Thus according to equation (19) and (20) one gets

$$\lambda = \frac{n_0^{-1}}{\rho} e^{-\frac{\mu_n \mu_0 e f}{2rV_0} Z} \quad (21)$$

Since the mass number A is related to the atomic number Z through the relation

$$A = Z + N$$

Where N is the number of neutrons therefore

$$Z = A - N \quad (22)$$

By setting

$$c_0 = \frac{\mu_n \mu_0 e f}{2rV_0} \quad (23)$$

Hence

$$\lambda_0 = n_0^{-1} \rho^{-1} e^{-c_0 Z}$$

The mean free path is thus given by

$$\lambda = n_0^{-1} \rho^{-1} e^{c_0 N} e^{-c_0 A} = \lambda_0 e^{-c_0 A} \quad (24)$$

Fig (7) shows the theoretical relation between  $\lambda$  and A graphically

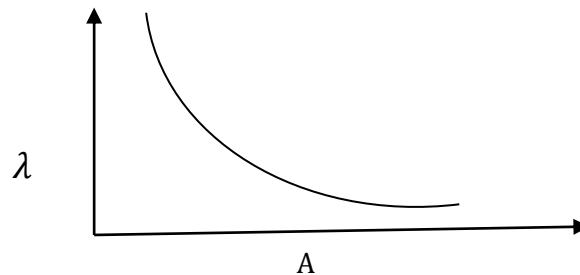


Fig (7) Theoretical relation of  $\lambda$  versus A

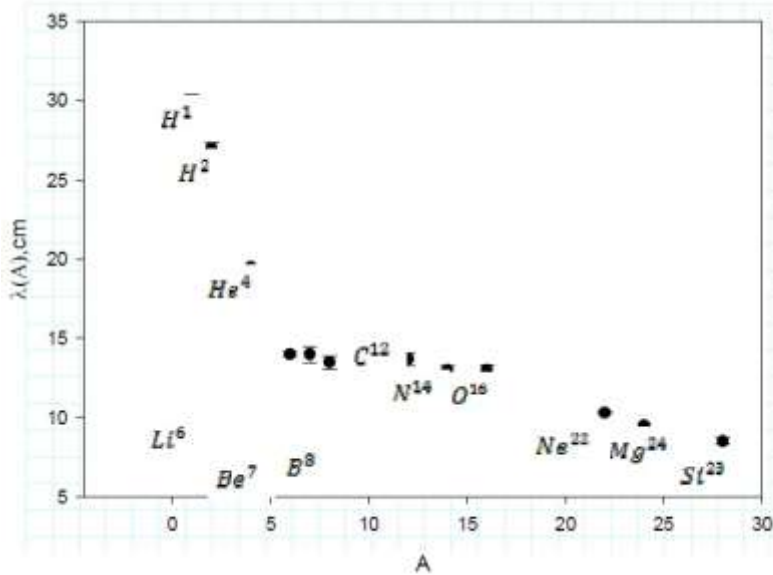


Fig (8) Empirical relation of  $\lambda$  versus  $A$

The theoretical curve in fig (7) can explain the empirical relation shown in fig (8) The empirical relations which relate the incident neutron energy  $E_n$  with the probability of fission  $P_f$  can be easily understood on the basis of energy concept The initial energy of fused nuclear  $E_f$  beside the energy  $E_n$  of the absorbed neutron contribute to the total energy  $E$  of fused nuclei. Thus the fusion probability is given by

$$P_f \sim n = n_0 e^{-\frac{E}{\bar{E}}} = n_0 e^{-\frac{E_f}{V_0}} e^{-\frac{E_n}{V_0}} \quad (25)$$

$$P_f = a_0 e^{-\frac{E_n}{V_0}} \quad (26)$$

Where the average energy of fused nuclei is assumed to result from mass energy

$$\bar{E} = V_0 = mc^2$$

This relation which is displayed graphically in Fig (9) Agrees with the empirical ones in Figs (10)

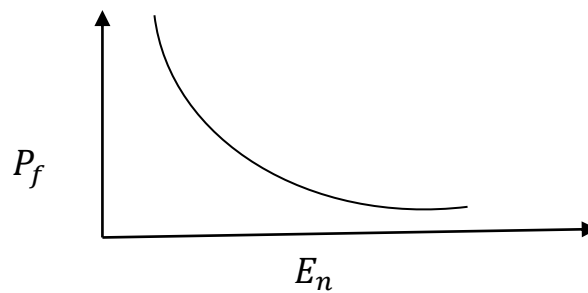


Fig (9) Theoretical relation between  $P_f$  and  $E_n$



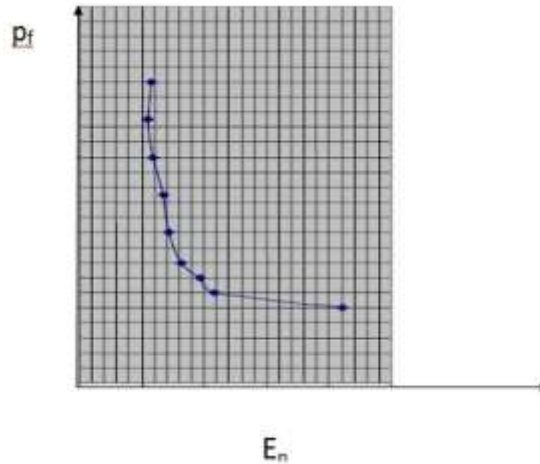


Fig (10) Empirical relation between  $P_f$  and  $E_n$

The increase of optical absorption by the nucleus can be also explained by assuming that  $n$  neutrons are absorbed by  $n$  nuclei per unit volume.

Thus the new mass of each nucleus  $m$  is equal to the original mass  $m_o$  beside the masses of the neutrons absorbed by one nucleus, i.e

$$m = m_o + \frac{n}{N}m_n \quad (27)$$

When a photon of energy  $E_i$  is incident on this nucleus it gives her kinetic energy.

$$K_n = \frac{1}{2}mv_o^2 \quad (28)$$

The photon new energy become

$$E = E_i - K_n = E_i - \frac{1}{2}\left(m_o + \frac{n}{N}m_n\right)v_o^2 \quad (29)$$

$$= E_i - K_o - a_o n$$

Where

$$K_o = \frac{1}{2}m_o v_o^2$$

$$a_o = \frac{m_n v_o^2}{2N} \quad (30)$$

Thus

$$I = I_o e^{\frac{E}{\bar{E}}} = I_o e^{\frac{E_i}{E_o}}$$

If the average photon energy is given by

$$\bar{E} = E_o$$

Thus the absorbed beam intensity is given by

$$I = I_o e^{\frac{E}{\bar{E}}} = I_o e^{\frac{E_i}{E_o} e^{\frac{k_o}{E_o} e^{\frac{a_o n}{E_o}}}}$$

$$I = b_o e^{c_o n} \quad (31)$$

Where

$$c_o = \frac{a_o}{E_o}$$

$$b_o = I_o e^{\frac{E_i}{E_o} e^{\frac{k_o}{E_o}}} \quad (32)$$

Fortunately the theoretical relation can easily explain the empirical relation in fig (11).Where the theoretical relation is displaying Fig (12) for small  $c_o$  equation (51) becomes

$$I = b_o(1 + c_o n) \quad (33)$$



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Let I  $b_o = 5$   $c_o = 1$

Then relation (33) is displayed graphically in fig (13)

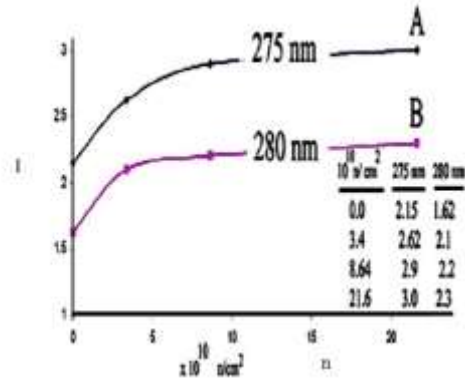


Fig (11) Empirical relation of I for photons versus n for neutron

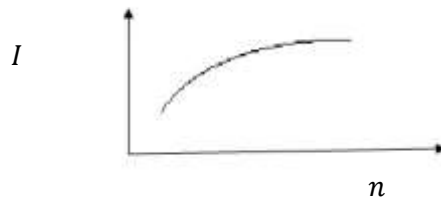


Fig (12) Theoretical relation of I for photons and n for neutron

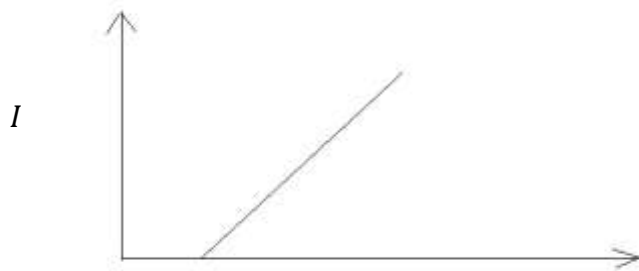


Fig (13) Theoretical relation of I and n

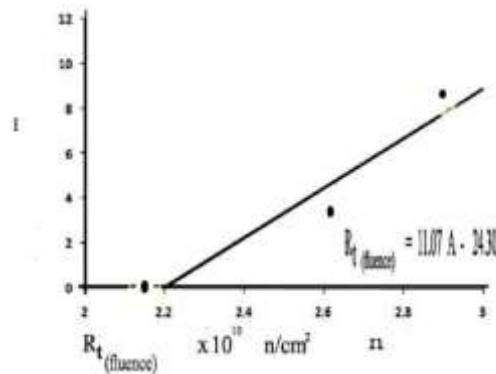


Fig (21) Empirical relation of  $I$  versus  $n$

This theoretical relation which is displayed in Fig (13) conforms with the empirical one in Fig (14) in the experiment done by H.O.Jubouri, Nada .F. Tawfiq and Ammar [4].

### III. DISCUSSION

The relation between found theoretically by using generalized Maxwell distribution (GMD) law (1). This relation is displayed graphically in Fig (1), by assuming that the scattered and incident fluxes obeyes Plank quantum energy relation the relation explains the experimental relations done by T.Abo-shdeed etal shown in fig (2).The relation between photon intensity  $I$  and path  $L$  is also displayed graphically in fig (3) by using (GMD) law, and assuming that the photon gain energy from all atoms on its path, like that happen in daman scattering. this agrees with the empirical one observed in in  ${}^9\text{Be} (p,n){}^9\text{B}$  scattering process which relates photon intensity  $I$  to its path  $L$  as show in fig (4).However the empirical relation which relates neutron intensity  $I$  to its path length  $L$  shows experiential increase c see fig (6) is  ${}^9\text{Be} (p,n){}^9\text{B}$  scattering process. This can be explained on the basis of (GMD) by assuming that the neutron in the flux is magnetically interact with all nuclei on its path as shown by equation (14) the relation displayed graphically in fig (5) conforms with the empirical relation in fig(6).The empirical relation between mean free path  $\lambda$  and mass number for  $H, He, C, N, O, Ne, Mg$  and  $Si$  shown in fig(8) can be also easily explained on the basis of GMD. This is done by assuming that  $Z$  protons revolve around the nucleus center to produce magnetic field. Then  $Z$  is related to the mass number  $A$  and intensity  $I$  according to equation (24). This theoretical relation is displayed graphically in fig (7) and conforms with the empirical one in fig (8).Finally GMD Law explains also the empirical relation between neutron energy  $E_n$  and fusion probability  $P_f$ .The theoretical relation in fig (9) is based on the assumption that the fused nuclei energy comes from the original nuclei energies beside neutrons energies. This relation in fig(9) agrees with the empirical on in fig (10).The empirical relation between intensity  $I$  and neutron flux  $n$  shown in Figures (11) and (14) can be also successfully explained by GMD assuming that some of neutrons are absorbed by nuclei targets which leads to increase in their mass (see Figures (12) and (13)).

### IV. CONCLUSION

The generalized Maxwell distribution Law can explain some neutron scattering processes easily by using simple mathematical relations.

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